fMRI BOLD signal decomposition using a multivariate low-rank model

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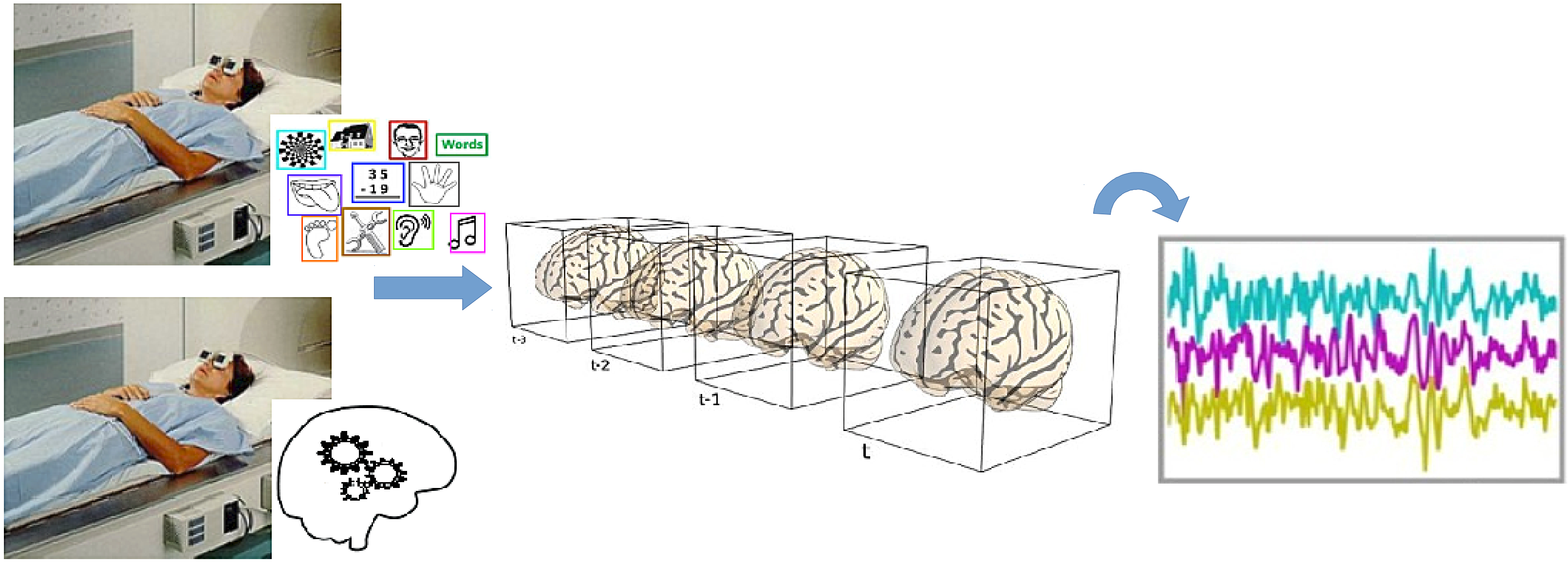
fMRI acquisition:

Task fMRI

Resting-state fMRI



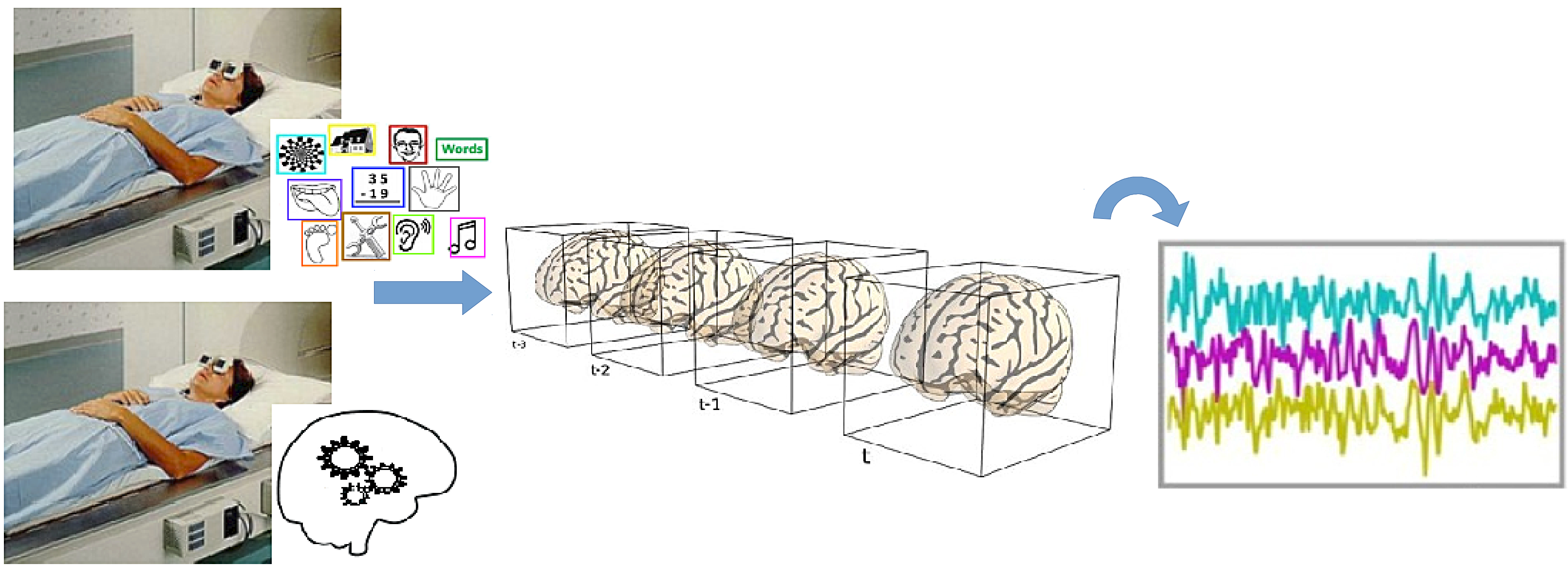
BOLD signal → **INDIRECT OBSERVATION OF NEURAL ACTIVITY**

[Ogawa et al, 1990, 1992]

fMRI acquisition:

Task fMRI

Resting-state fMRI



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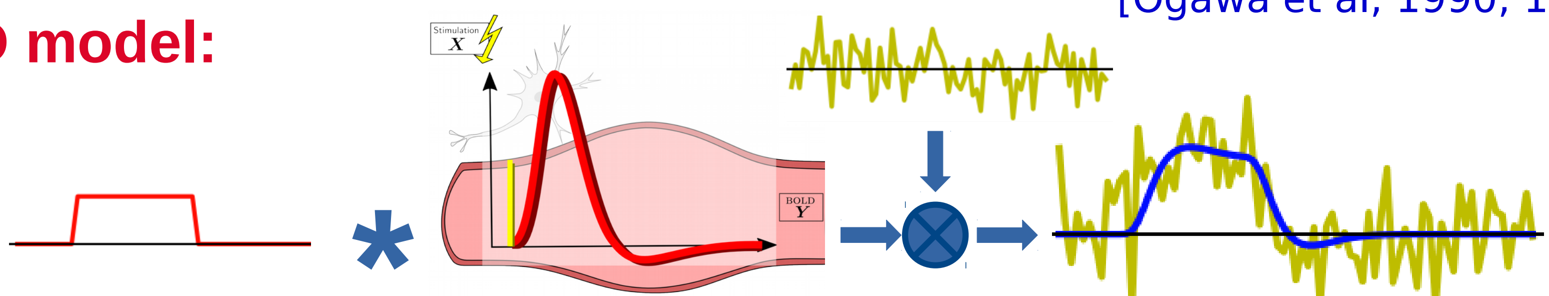
BOLD model:



Neural activation signal
Z

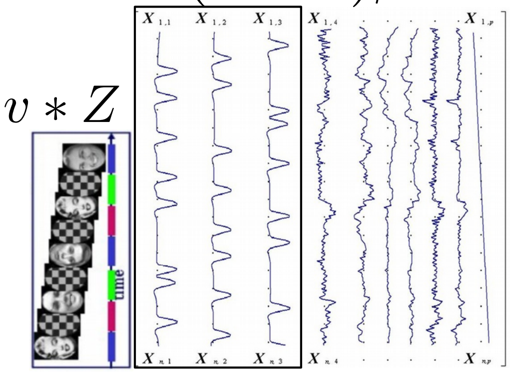
Haemodynamic Response Function (HRF)
V

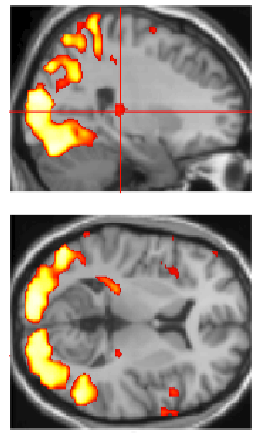
Observed time-courses (BOLD signal)
Y

CLASSICAL ANALYSIS PIPELINE

Task fMRI:

$$Y = (v * Z)\beta + E$$





[Poldrack et al, Handbook of Functional MRI Data Analysis, 2013]

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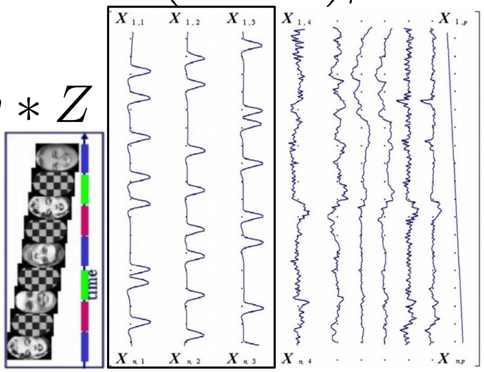
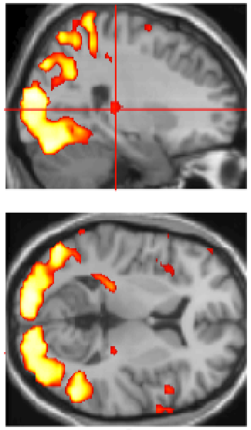
$$Y = (v * Z)\beta + E$$

$v * Z$





Univariate (voxelwise) & Explicit paradigm

[Poldrack et al, Handbook of Functional MRI Data Analysis, 2013]

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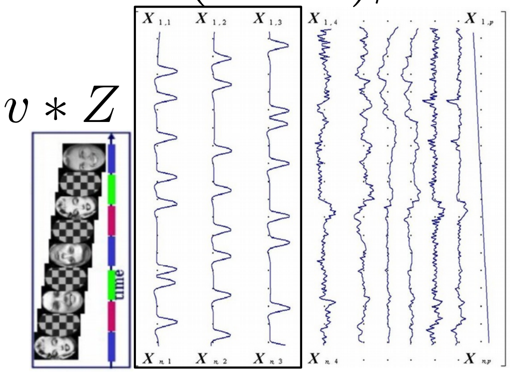

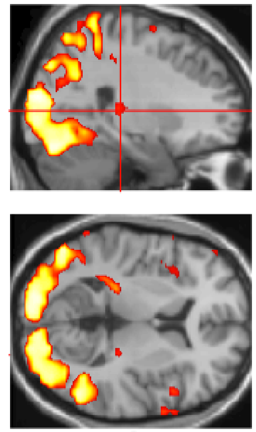

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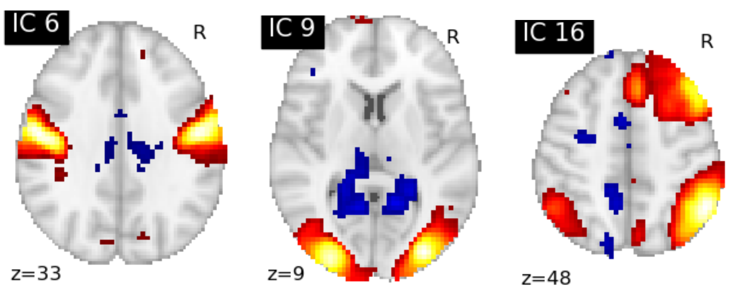
Univariate (voxelwise) & Explicit paradigm

Resting-state fMRI:



3 independent components of the ICA on resting-state data (ADHD dataset)

[Varoquaux et al, 2009]

CLASSICAL ANALYSIS PIPELINE

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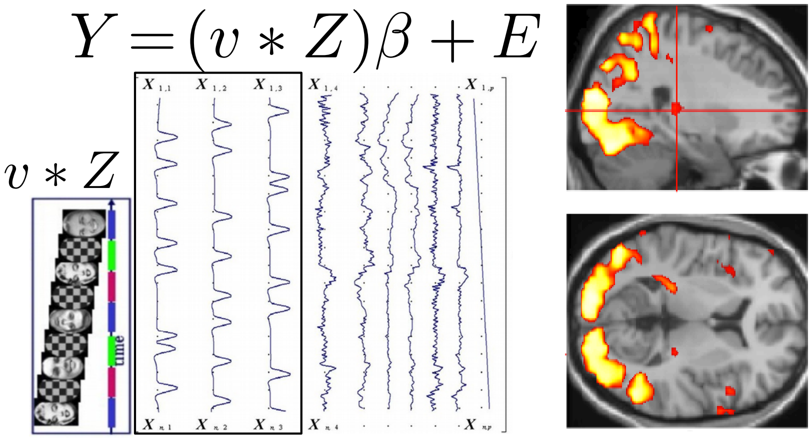

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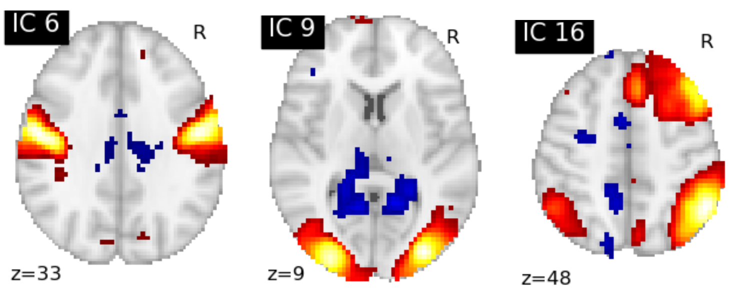

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Multivariate & Paradigm free

LIMITATION OF CLASSICAL ANALYSIS PIPELINE

Our objective:

- Develop a multivariate deconvolution approach for functional connectivity analysis of **neural activation** signals

➔ Accommodate both task and rs-fMRI data

- Multivariate extension of the 'Total Activation' framework [Karahanoglu et al, 2013]

OBSERVATION MODEL OF THE BOLD DATA

Model: multivariate approach (temporal components and corresponding spatial maps) with predefined HRF

$$Y = \sum_{k=1}^{K} {u_k}^\top (v * z_k) + E$$

P number of voxels
L HRF length
T number of scans
$\widetilde{T} = T - L + 1$ the number of time points in the temporal atoms
K the number of atoms (model rank)
E additive Gaussian noise

Parameters to estimate: z_k, u_k

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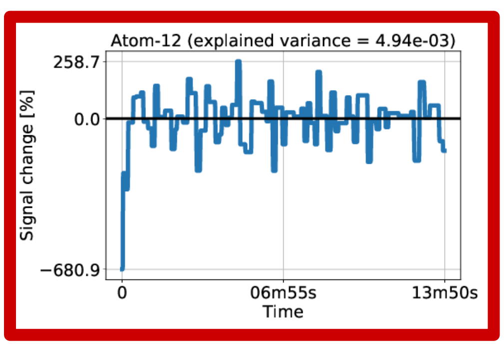
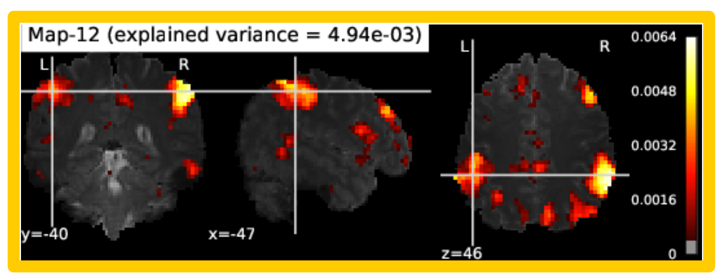
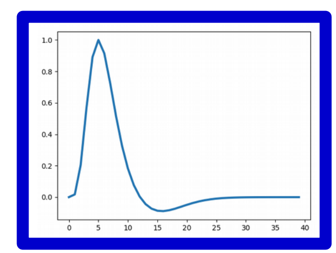
$\mathbf{z_k}$



$\mathbf{u_k}$



$\mathbf{v}$



[Friston et al, 2000]

MULTIVARIATE DECONVOLUTION OF THE BOLD DATA

Spatio-temporal constrained optimization problem:

$$J((u_k)_k, (z_k)_k) = \frac{1}{2} \left\| Y - \sum_{k=1}^{K} {u_k}^\top (v * z_k) \right\|_F^2 + \lambda \sum_{k=1}^{K} \|D z_k\|_1$$
$$\text{subject to} \quad \|u_k\|_1 = \eta \quad \text{and} \quad u_{jk} \geq 0 \qquad (1)$$

- **Data fidelity term:**
  - The Gaussian noise leads to a quadratic loss
- **Temporal components:**
  - TV regularization: promote sparsity of the 1st order derivative [Karahanoglu et al, 2013]
- **Spatial constraints:**
  - Positivity of each entry in each spatial map to avoid sign ambiguity with the corresponding temporal component
  - L1 norm of each spatial map fixed to a certain level to avoid any scale ambiguity

MULTIVARIATE DECONVOLUTION OF THE BOLD DATA

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$$\text{subject to} \quad \|u_k\|_1 = \eta \quad \text{and} \quad u_{jk} \geq 0 \qquad (1)$$

- **Strategy of minimization:**
  - The global cost function is bi-convex in (**z_k**, **u_k**): Each sub-problem is convex
  - We propose to alternate the minimization between the **z_k** and the **u_k**.
  - The **z_k** are initialized to zero and **u_k** to a truncated Gaussian random vector (to ensure positive values for **u_k**)

STRATEGY OF MINIMIZATION

Minimization of the cost-function: different possible approaches

- **z_k estimation step:**

$$\boxed{\mathrm{J}_z((z_k)_k) = \frac{1}{2}\left\|Y - \sum_{k=1}^{K} u_k^\top (v * z_k)\right\|_F^2 + \lambda \sum_{k=1}^{K} \|D z_k\|_1}$$

Analysis formulation

ISTA [Daubechies et al. 2004]
FISTA [Beck, Teboulle, 2009]
Restarting-FISTA [Liang et al, 2013]
Greedy FISTA [Liang et al, 2013]
Condat-Vu [Condat, 2016]

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Condat-Vu [Condat, 2016]

$$\mathrm{J}'_z((z_k)_k) = \frac{1}{2}\left\| Y - \sum_{k=1}^{K} u_k^\top (v * L z_k) \right\|_F^2 + \lambda \sum_{k=1}^{K} \|z_k\|_1$$

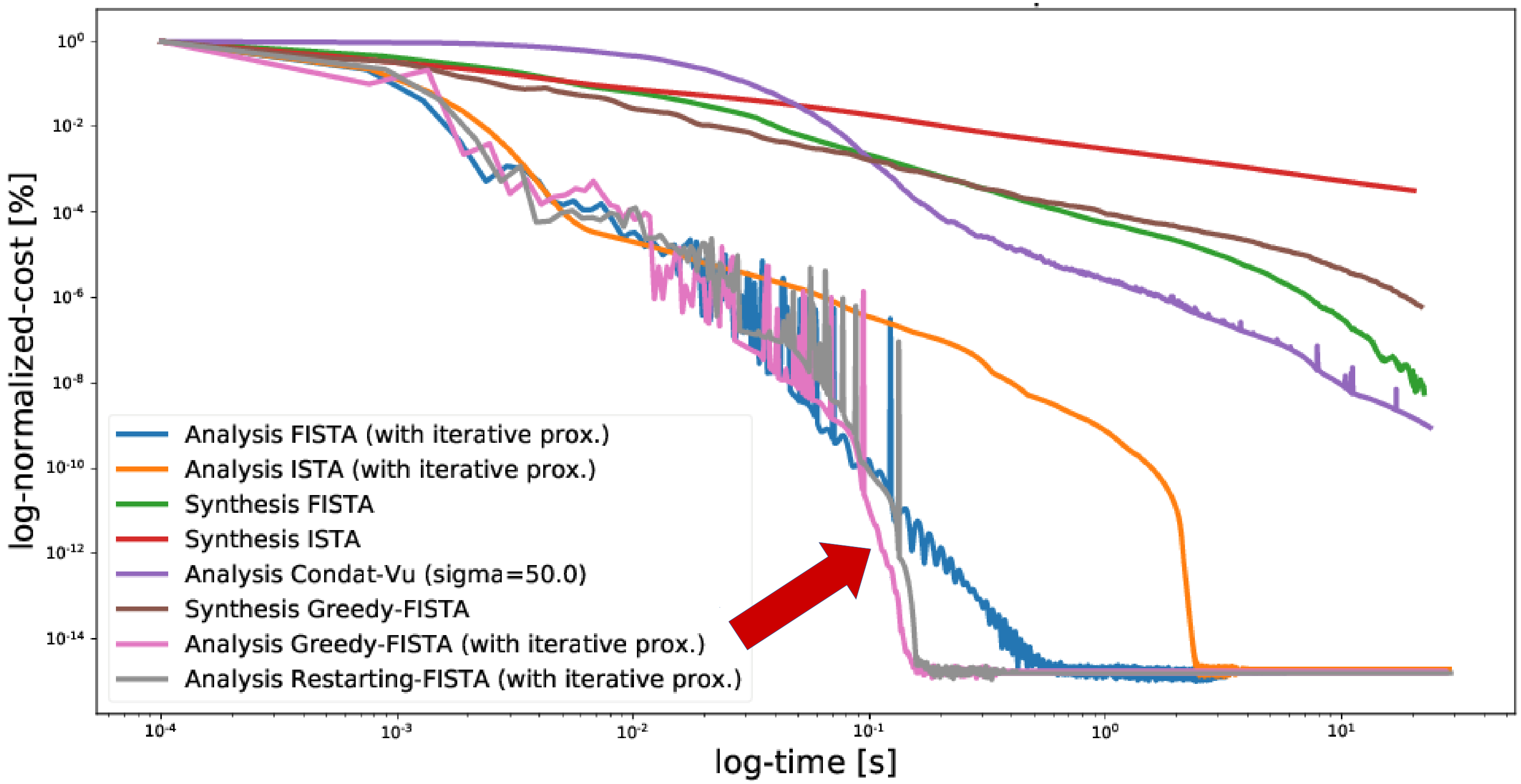
Synthesis formulation

ISTA [Daubechies et al. 2004]
FISTA [Beck, Teboulle, 2009]
Restarting-FISTA [Liang et al, 2013]
Greedy FISTA [Liang et al, 2013]

CONVERGENCE RATE COMPARISON FOR THE RECOVERY OF NEURAL ACTIVATION SIGNALS

Convergence rate comparison:

Convergence rate comparison for the temporal components estimation problem



STRATEGY OF MINIMIZATION

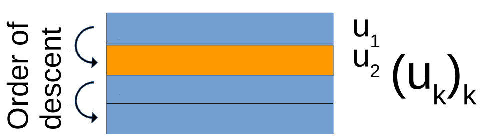
Minimization of the cost-function: different possible approaches

- **u_k estimation step:**

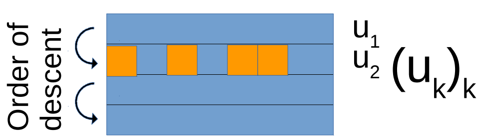
$$\boxed{\begin{aligned} J_u((u_k)_k) &= \frac{1}{2}\left\| Y - \sum_{k=1}^{K} u_k{}^{\top}(v * z_k) \right\|_F^2 \\ \text{subject to} \quad \|u_k\|_1 &= \eta \quad \text{and} \quad u_{jk} \geq 0 \end{aligned}} \qquad (1)$$

ISTA, FISTA, Greedy FISTA: (cf. previous slide)

Mairal: coordinate descent with optimal step-size in the context of online learning [Mairal et al, 2009]

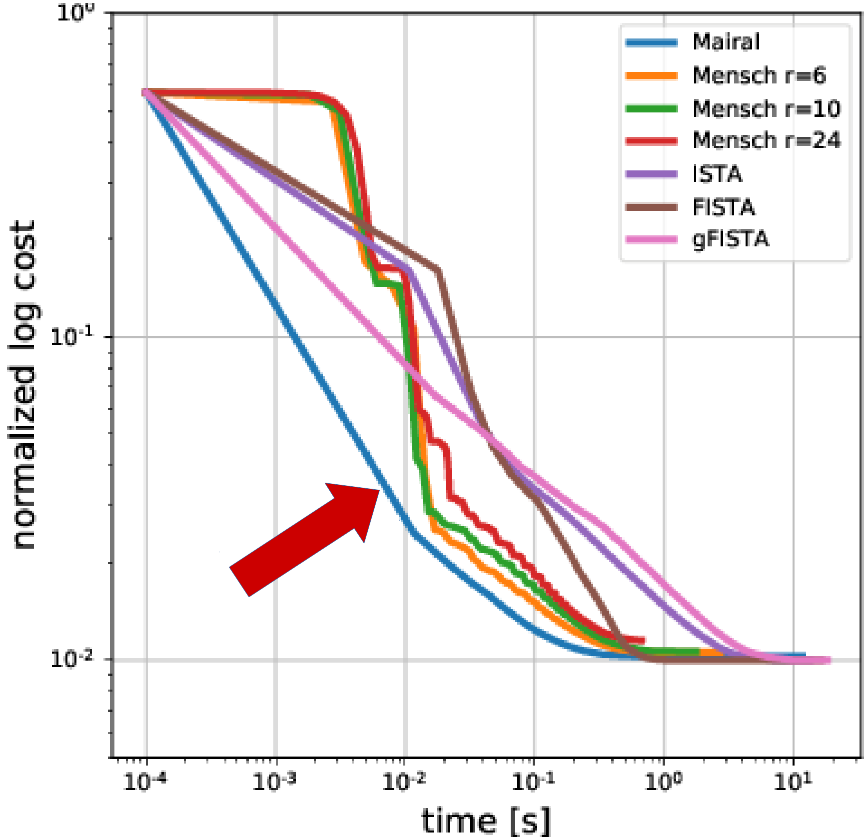


Mensch: based on Mairal's algorithm with a subsampling along a dimension of the problem [Mensch et al, 2016]



Convergence rate comparison:

Convergence rate comparison for the spatial maps estimation problem



THE PROPOSED ALGORITHM

The final algorithm:

Algorithm 1: Low rank decomposition of the BOLD signal.

Input: BOLD signal $\boldsymbol{Y}$, ϵ

initialization: $\boldsymbol{z}_k^{(0)} = \mathbf{0}_{\widetilde{T}}$, $\boldsymbol{u}_k^{(0)} = \boldsymbol{u}_k^{(init)}$, $i = 1$;

repeat

Estimate the temporal atoms $\boldsymbol{z}_k^{(i)}$ with fixed $\boldsymbol{u}_k^{(i-1)}$:

$$\arg\min_{(\boldsymbol{z}_k)_k} \frac{1}{2} \left\| \boldsymbol{Y} - \sum_{k=1}^{K} \boldsymbol{u}_k^{(i-1)\top} (v * \boldsymbol{z}_k) \right\|_F^2 + \lambda \sum_{k=1}^{K} \|\boldsymbol{D} z_k\|_1$$

Restarting-FISTA

Estimate the spatial maps $\boldsymbol{u}_k^{(i)}$ with fixed $\boldsymbol{z}_k^{(i)}$:

$$\arg\min_{(\boldsymbol{u}_k)_k} \frac{1}{2} \left\| \boldsymbol{Y} - \sum_{k=1}^{K} \boldsymbol{u}_k^{\top} (v * \boldsymbol{z}_k^{(i)}) \right\|_F^2$$

$$\text{subject to} \quad \|\boldsymbol{u}_k\|_1 = \eta \quad \text{and} \quad u_{kj} \geq 0$$

Mairal's algorithm

until $\frac{J((\boldsymbol{z}_k^{(i-1)})_k, (\boldsymbol{u}_k^{(i-1)})_k) - J((\boldsymbol{z}_k^{(i)})_k, (\boldsymbol{u}_k^{(i)})_k)}{J((\boldsymbol{z}_k^{(i-1)})_k, (\boldsymbol{u}_k^{(i-1)})_k)} \leq \epsilon$;

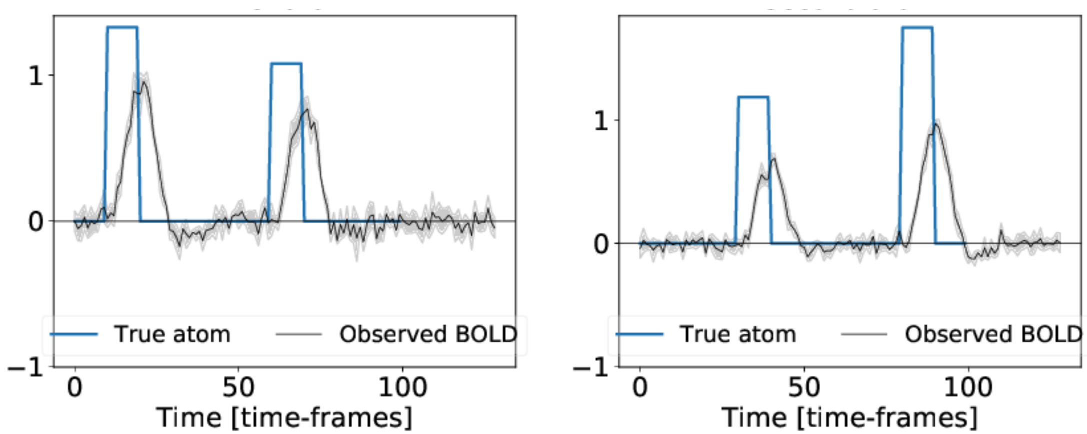
SIMULATED DATA DECOMPOSITION

Simulated paradigm: two activation blocks

Data:

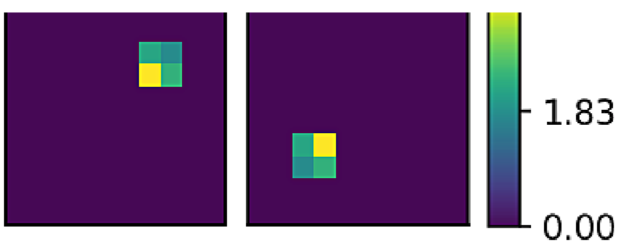
- $K_{true} = 2$
- T = 100
- TR = 1.0s
- P = 100
- Each true spatial maps contains a single square regions of 'activity'
- Signal-to-noise ratio: 0.1, 0.5, 1.0, 5.0, 10.0, 15.0, 20.0 dB $\quad \mathrm{SNR} = 10 \log_{10}\left(\frac{\|\sum_{k=1}^{K} u_k^\top (v * z_k)\|_2^2}{\|E\|_2^2}\right)$
- Each temporal component contains *2 blocks* whose duration was fixed to *10 s* and the magnitude was randomly drawn from a Gaussian distribution centred on 1.0.

$\mathbf{z}_{true}$



In **blue**, the true temporal atoms
In **black**, the observed BOLD signal (*here* SNR = 1.0 dB)
In grey, the standard deviation across voxels encoded by transparency around mean curves

$\mathbf{u}_{true}$



In yellow-**purple** maps define the spatial ground truth

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- Each temporal component contains *2 blocks* whose duration was fixed to *10 s* and the magnitude was randomly drawn from a Gaussian distribution centred on 1.0.

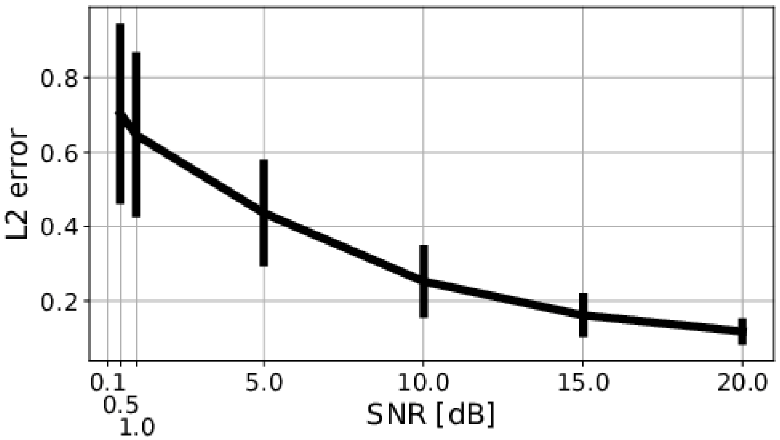
Algorithm parameters for estimation:

- K = 2
- η = 10.0
- λ = grid-search for each SNR scenario
- Max-iteration = 30
- 3 initializations tested

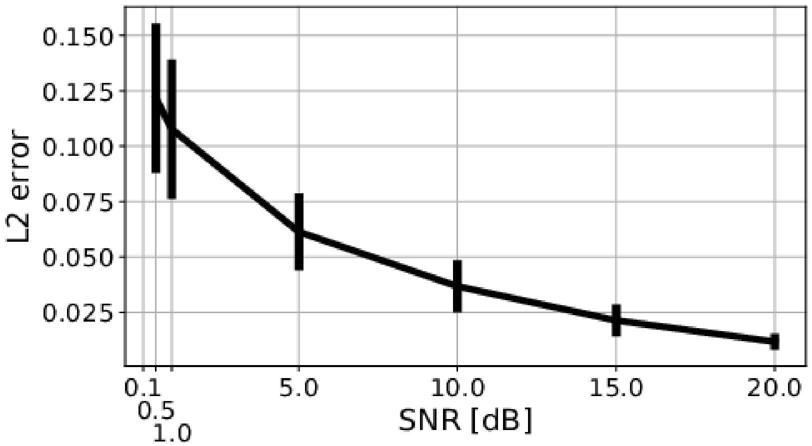
SIMULATED DATA DECOMPOSITION

Simulation: two activation blocks case

(z_k, u_k) Evolution of the estimation error



Evolution of the z_k-estimation error w.r.t the SNR



Evolution of the u_k-estimation error w.r.t the SNR

➔ The estimation error decreases while the SNR increases

REAL MOTOR TASK DATA DECOMPOSITION

Motor task: Human Connectome Project (**HCP**) dataset

Data:

- HCP release: HCP-1200 [www.humanconnectome.org]
- Motor task fMRI data
- One subject (randomly chosen)
- ~3min30s of acquisition
- Spatial resolution: 2x2x2mm
- P = 57790
- T = 284
- TR = 0.72s

Data were provided (in part) by the Human Connectome Project, WU-Minn Consortium (Principal Investigators: David Van Essen and Kamil Ugurbil; 1U54MH091657) funded by the 16 NIH Institutes and Centers that support the NIH Blueprint for Neuroscience Research; and by the McDonnell Center for Systems Neuroscience at Washington University.

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Motor task:

- Each condition were preceded by a *visual cue* of 3 s
- The motor tasks consisted of a sequence of *right/left hands clenching* and *right/left foot squeezing*
- Each condition lasted 12 s

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Motor task:

- Each condition were preceded by a *visual cue* of 3 s
- The motor tasks consisted of a sequence of *right/left hands clenching* and *right/left foot squeezing*
- Each condition lasted 12 s

Decomposition parameters:

- K = 40
- η = 10.0
- λ = 1.0e-2
- Max-iteration = 30
- 3 initializations tested

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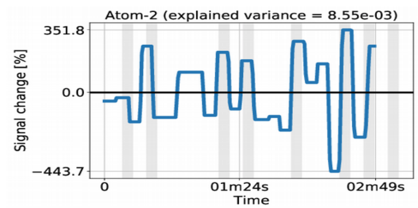
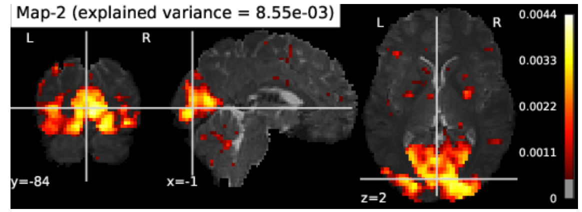
REAL MOTOR TASK DATA DECOMPOSITION

Motor task: Human Connectome Project (**HCP**) dataset

Temporal activation **10%-thresholded spatial map**

Component #2

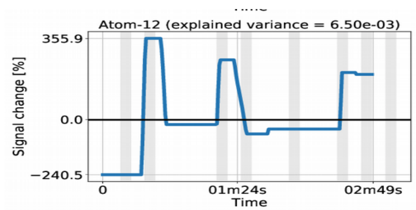


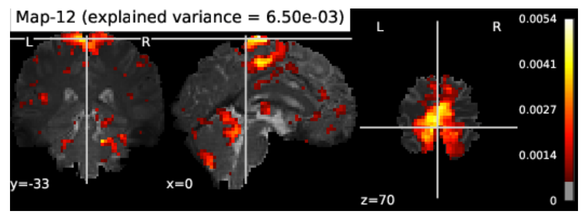


Visual cortex

Each motor task was cued by a visual instruction

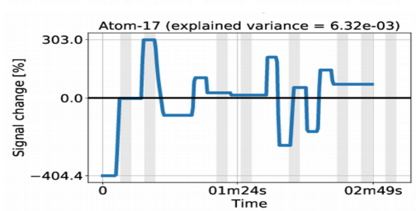
Component #12

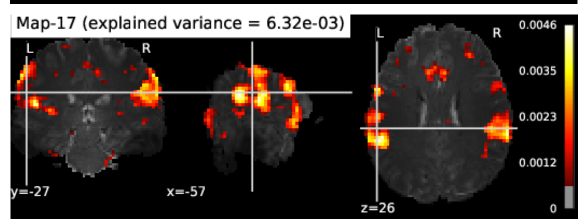




Primary somatosensory cortex

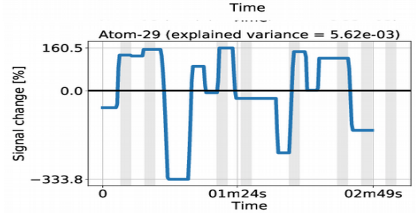
Component #17

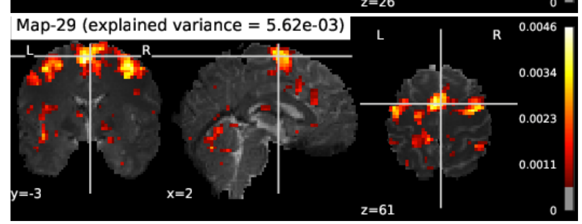




Primary motor cortex

Component #29





Supplementary motor area

c.t = 33 min

REAL RESTING-STATE DATA DECOMPOSITION

Resting state: Human Connectome Project (**HCP**) dataset

Data:

- HCP release: HCP-1200 [www.humanconnectome.org]
- Resting-state fMRI data
- One subject (randomly chosen)
- ~14min of acquisition
- Spatial resolution: 2x2x2mm
- P = 57790
- T = 1200
- TR = 0.72s

Decomposition parameters:

- K = 10
- η = 10.0
- λ = 5.0e-3
- Max-iteration = 30
- 3 initializations tested

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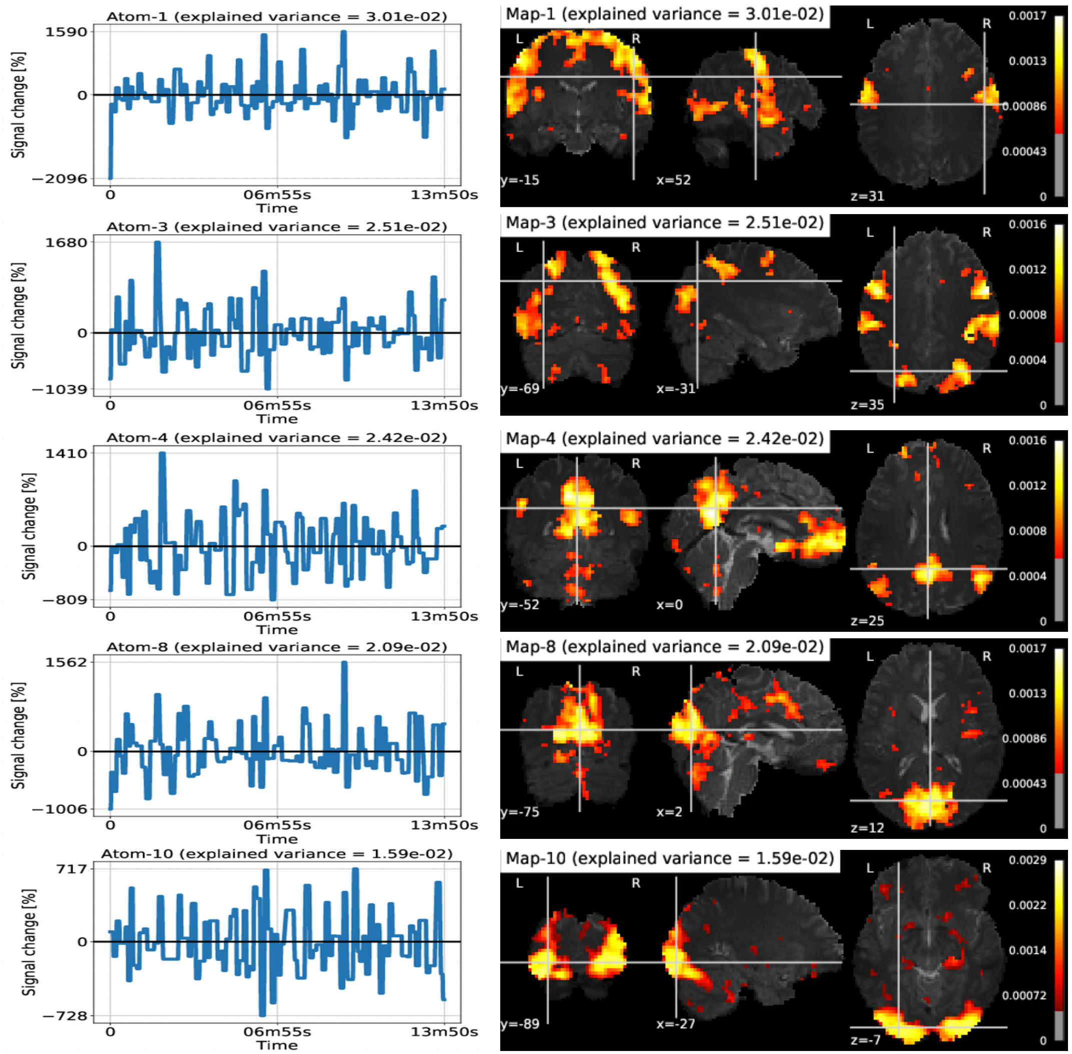
REAL RESTING-STATE DATA DECOMPOSITION

Resting state: Human Connectome Project (**HCP**) dataset

Temporal activation **10%-thresholded spatial map**



Component #1 — **Motor cortex**

Component #3 — **Attention network**

Component #4 — **Default Mode Network**

Component #8 — **Visual cortex (V2)**

Component #10 — **Visual cortex (V1)**

c.t = 6 min

REAL RESTING-STATE DATA DECOMPOSITION

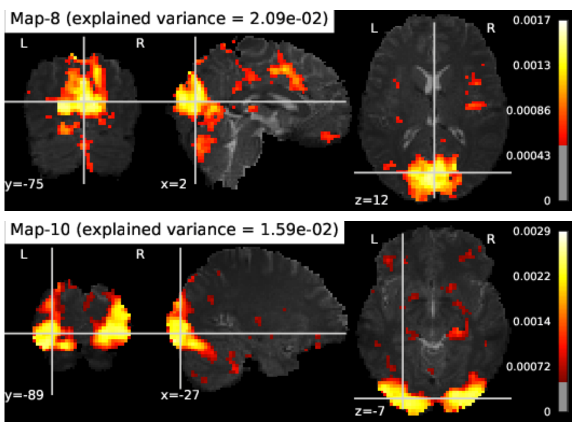
Resting state: Human Connectome Project (**HCP**) dataset

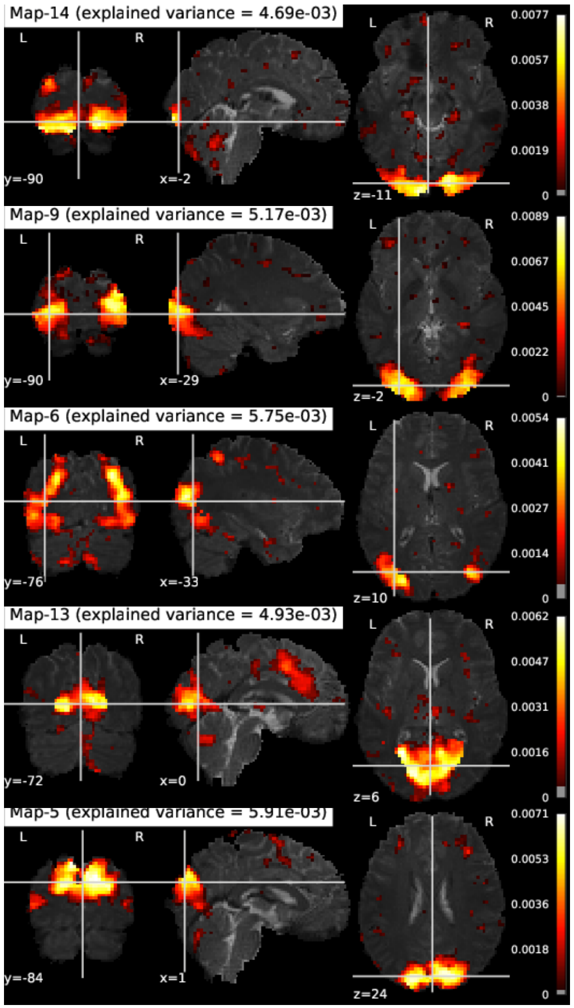
K = 10
10%-thresholded spatial map

K = 40
10%-thresholded spatial map

Visual cortex (V2)

Visual cortex (V1)





c.t = 38 min

REAL RESTING-STATE DATA DECOMPOSITION

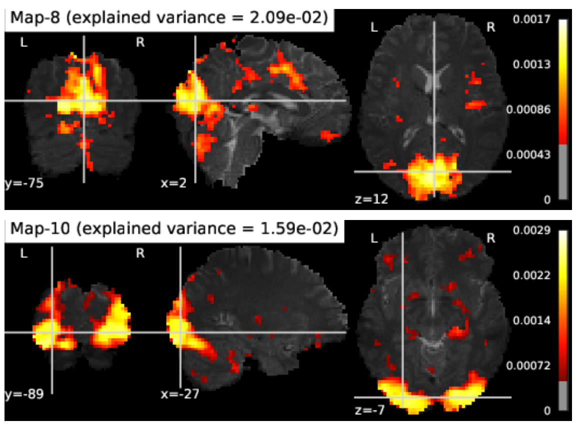
Resting state: Human Connectome Project (**HCP**) dataset

K = 10

10%-thresholded spatial map

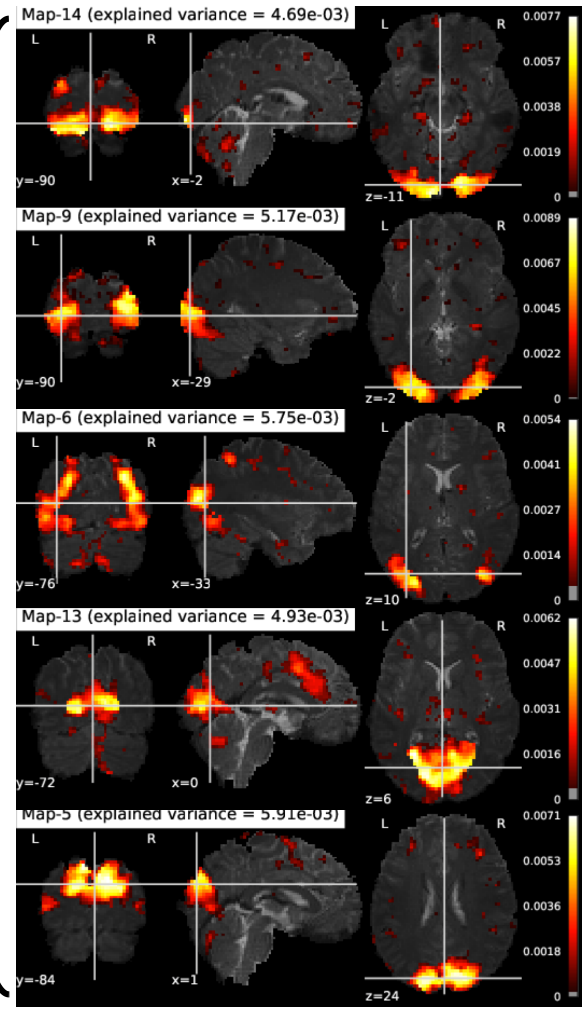
Visual cortex (V2)

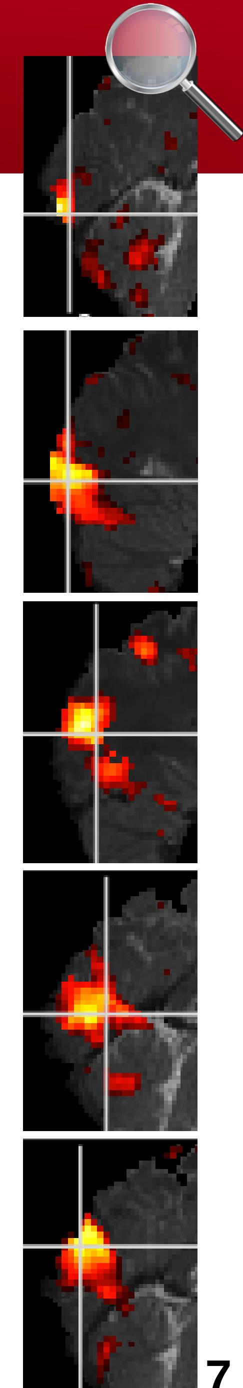
Visual cortex (V1)



K = 40

10%-thresholded spatial map





c.t = 38 min

CONCLUSION

Summary:

- We provided a new low-rank decomposition of the BOLD signal which yields deconvolved neural activity signals and their corresponding spatial maps
- The proposed algorithm performs this decomposition in a reasonable computing time
- We showed that our method provides meaningful decomposition on the neural activity in resting-state and task fMRI

Future works:

- Blind deconvolution: estimate one HRF for each predefined brain region
- Unsupervised estimation: estimate λ, K (model comparison: r^2 score, etc)
- Characterize the statistical properties of the decomposition (neural activity signals)
- Validation on large scale datasets (HCP, Synchropioïd, etc)

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GitHub https://github.com/CherkaouiHamza/seven

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